

A STUDY TO OPTIMIZE THE RESOURCE ALLOCATION OF EMERGENCY DEPARTMENT USING QUEUING THEORY

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Abstract

This paper aims to study the problems occurring in Emergency Departments of various private and public Hospitals. Because of adverse environment conditions, health problems are increasing day by day with increasing population. Patients arriving in Emergency Department needs timely care but it is usually seen that Patients needs to wait for long times. Admitting a patient anyhow in Emergency Department is the main motive of the hospital. A Survey was conducted to know the patient's satisfaction rate in various Emergency Departments. The main problem faced by the patients is delay in getting service because of non availability of idle resources to operate. Improving the service of Quality provided by of Emergency Department and reducing waiting time will require some extra investment. Optimizing the resource allocation and minimizing the service cost is the main motive of this work. This paper will help the Hospital Management teams to improve the quality service of Emergency Departments.

Keywords: *Queuing Theory, Healthcare Management, Emergency Department.*

Introduction:

Queuing theory is a branch of mathematics that studies and models the act of waiting in lines. Waiting lines or queues can be seen in any system where arrival rate is more than the system can handle. The first paper on queuing theory, "The Theory of Probabilities and Telephone Conversations" was published in 1909 by A.K. Erlang. No doubt that queuing theory was first applied to analyze the problem of telephone waiting calls by A.K. Erlang in 1913 but nowadays everybody is becoming aware of the applications of queuing theory in other areas. In developing a solution to this problem he began to realize that the problem of minimizing waiting time was applicable to many fields, and began developing the theory further. Queuing models are used for estimating waiting time of a patient, utilization of service, models system design, and models for evaluating appointment systems. A queuing system helps minimizing the waiting time of patients and

maximizing the utilization of the servers i.e. doctors, nurses, hospital beds etc. Queuing is not new but recently hospitals have begun to use it effectively. After knowing the use of queuing models in determining the optimal allocation of resources in a hospital, hospital management teams are using it more and more. The need of applying queuing theory and queuing models in healthcare settings is because it can help to save someone's life.

Although there are different levels of healthcare centers but they all act in a similar fashion in terms of treating a patient. They all have a set of activities that every patient needs to undergo during the treatment. Hospital is a very important section of healthcare and has a major impact on disease prevention, early detection treatment (Akbari 2016). Moreover the rate of patient's admitting in public hospitals is more as compared to private hospitals because service provided in public hospitals is cheaper than that of private hospitals. With increasing population, admission in Emergency Departments is also increasing but due to limited amount of resources and facilities long waiting lines are seen. The goal is to provide some solution to these waiting lines, delays. Therefore optimizing the use of Resources and providing better service is main motive of this study. Managing patients waiting time, providing better service, optimizing the allocation of resources available is a big challenge for public hospitals all over the world. Long queues in the emergency departments may have negative impact on the hospital reputation. These queues are formed because of random arrivals of patients in hospitals. Because of complex and variable nature of patient's arrival it is very difficult to solve the problem without using new technologies. Using queuing theory and queuing models, it is possible to calculate the exact amount of resources required in a hospital.

Emergency Department can be understood as a network of queues where patients arrive, wait, get service and then go to home or get admitted to the

hospital (Vass and Szabo, 2015). The main motive of queuing theory is to make a mathematical model of patients needing service and use that model to calculate the number of resources required such as nurses, doctors, beds, medical equipment etc (Malik and Belwal 2016).

Although queuing theory has a lot of applications but it has been used widely in service industries (Nosek and Wilson, 2001). They have used queuing theory in pharmacy area with special focus to improve customer's satisfaction.

Literature review:

Samuel Fomundam and Jeffrey Herrmann (2007) have done a survey of applications of queuing theory in healthcare. They applied queuing theory and queuing models in systems at different scales that includes small individual units, hospitals and regional healthcare centers. They have used queuing theory to study system design, appointment system, waiting time and utilization analysis.

S. Creemers et al (2007) have described the impact of sudden and unplanned interrupts and absences of patients in a waiting list in [2]. They have used the concept of queuing models to reduce delay in a healthcare system.

Lakshmi C et al (2013) have contributed their research in the applications of queuing theory in healthcare. In their work they examined digital libraries, journals, books, conference papers etc. They concluded that most of the articles are published in or after 2000 and all this is because of advancement in technology.

Fatma Poni Mardiah et al (2013) have analyzed data to study appointment system in a public hospital of Indonesia. Moreover their contribution is in reducing outpatient's waiting time in a hospital. In their study, they have concluded that hospital management team should focus on the four main quality indicating parameters of a healthcare system naming appointment system, waiting time and patient flow and capacity. Also they have explained the reasons why people prefer to go in a private hospital rather than government hospitals.

Mohammadkarim Bahadori et al (2014) have analyzed a hospital pharmacy

performance using queuing theory and simulation model. This study was conducted in military hospital in Iran; Tehran shows that queuing theory can act as a powerful tool to manage a pharmacy. They collected data regarding the time of filling prescription drugs for patients coming in morning and evening shifts using the preplanned forms. Collected data was further analyzed by queuing models. Moreover they analyzed the data collected for both morning and evening shifts and concluded that patient's waiting time can be reduced by assigning job to multitasking persons.

Olorunsola S. Aetal (2014) considered the queuing analysis of patient flow in a hospital. Patient flow refers to progress of a patient's health status (Olorunsola et al 2014). They calculated various parameters like bed occupancy rate, length of stay distribution using M/M/C model. Finding this optimal number of beds in a hospital is a very common practice to enhance the service quality of a hospital. Using these parameters hospital managers can improve the service quality. They also explained how optimal number of beds effect patient flow in a hospital.

Hajnal Vass et al (2015) have studied the concept of patient flow in an Emergency Department. This study was conducted in Romania. They have used M/M/c model to study the quality indicating parameters of the Emergency Department situated in Romania and reveals that how queuing theory can help in optimal allocation of resources in an Emergency Department. Moreover they also analyzed that increasing the quality of service provided by the hospital will increase the cost but it is important to optimize the whole process.

McClain (1976) reviews research on applying queuing models to study the impact of bed assignment policies on utilization, waiting time and the probability of turning away patients.

Nkeiruka Amah et al (2013) have done a survey to find patient's satisfaction rate in Nigerian hospital. They also find out the various causes of long waiting lines in

hospitals. They calculated patient's satisfaction rate by having a questionnaire.

Data Collection

This study was conducted in an Emergency Department of a government hospital situated in palwal. This Emergency Department is one of the cheapest Emergency Department among the hospitals of the city. Area of this Emergency Department is approximately 3000 square meters. Emergency Department of this hospital works for seven days in a week and twenty four hours a day.

Emergency Department of this hospital has following units:

- Resuscitation Area
- Major Trauma / Medical Areas
- Consultation Rooms
- Patient Waiting Area
- Minor Procedure Rooms
- Major Operating Room
- Observation Units
- Injection Room
- Immediate care unit
- Pediatric Emergencies
- Computer tomography
- Triage area

This Emergency Department was observed for 5 months from October 2018 to February 2019 and a total of 2946 patients were registered. It was found that patients arrival rate on weekends (Saturday and Sunday) is more as compared to other weekdays. One more thing that was noticed is that patient's arrival rate increases after 7 p.m and most of the patients coming in this duration are of RTA (Road Traffic Accidents).

Questionnaires were conducted in order to find the patient's satisfaction rate. Also the patient's suggestions to improve the quality of service of the Emergency Department were recorded. Some of the suggestions include increasing the number of doctors, doctors should come on time, more beds are required to reduce the delay in getting service, hospital should follow FCFS rule for admitting the patients until an emergency required.

Hospital records shows that when they diagnosis the reason behind the overcrowding of patients of RTA in night, maximum of the patients are of drink

and drive. Following table1 shows the number of arrival of patients of different types of emergencies.

Table1

Type of Emergency	Number of Patients	Percentage
RTA	951	32.28
Burn cases	314	10.65
Poisson cases	712	24.16
Hypertension	475	16.12
Fever	232	7.87
Injury	108	3.66
Heart attack	98	3.32
Others	56	1.90
Total	2946	100

Research will focus on busy hours of the Emergency Department. The data collected from the hospital information system was imprecise in some parameters such as patient's length of stay .Therefore data is collected from the available information in the hospital as well as by sampling.All patients who entered in Emergency Department whether they require urgent service or not were included in the study. Information regarding patient's arrival and patients going to various units of Emergency Department was collected from the hospital database.

Methods and calculations:

Retrospective data was collected from an Emergency Department by using the already existing data bases in the hospital. Moreover some of the data was collected by sampling because the study cannot be totally determined by already existing records as the records obtained may not be true in complete sense. Some of the incomplete records of patients are excluded from the study.This Emergency Department works in a systematic way. First of all patient arrived in waiting area is directed to triage area where some doctor evaluates triage level of the patient (risk of the disease).Then as per triage level patient is moved to various needed departments.

In order to reduce the consequences of delays on the people admitted in Emergency Department, the literature in this area was studied. Moreover hospital situations were analyzed with special focus on Emergency Department and necessary information (number of beds, number of nurses, number of doctors and amount of resources available) was collected. Frequent causes of accidents, type of emergencies and reasons of overcrowding in Emergency Department were evaluated.

The study has used M/M/C: FCFS/ ∞/∞ , where
 M=Markovian (or Poisson) arrivals and exponential service time
 c=Number of servers
 FCFS = First come, first served
 ∞ = Infinite system limit; ∞ = Infinite source limit.
 Patient's arrival time follows Poisson distribution and service time follows exponential distribution.
 Moreover let us assume that $\lambda/c\mu < 1$.

Formulas and notations to be used are as follows:
 λ =Arrival rate
 μ = Service rate
 c=Number of servers
 ρ =system utilization
 P_0 =Probability that there is no patient in the system
 P_k =Probability that there are k patients in the system and is given by
 $P_k = P_0(\lambda/\mu)^k(1/k!)$ $k < c$
 $P_k = p_0(\lambda/\mu)^k(1/c!c^{k-c})$ $k \geq c$

Calculations:

On the basis of data collected we have found that
 $\lambda = 9$ per hour means that 9 patients are arriving to Emergency Department in one hour.
 $\mu = 4$ per hour means that 4 patients are getting getting service in an hour.
 $c = 4$ means 4 physicans or human resources are there to handle the patients.
 $\rho = \lambda/\mu = 9/4$

We know that total sum of probabilities must be equal to one. Therefore we have
 $P_0 + P_1 + P_2 + \dots = 1$
 $P_0 + P_0(\lambda/\mu) + P_0(\lambda/\mu)^2(1/2!) + P_0(\lambda/\mu)^3(1/3!) + P_0(\lambda/\mu)^4(1/4!) + P_0(\lambda/\mu)^5(1/4!) + \dots = 1$ - eq(1)
 Putting the values in equation 1 we get
 $P_0 + P_0(9/4) + P_0(9/4)^2(1/2!) + P_0(9/4)^3(1/3!) + P_0(9/4)^4(1/4!) + P_0(9/4)^5(1/4!) + \dots = 1$
 $P_0[1 + (9/4) + (9/4)^2(1/2!) + (9/4)^3(1/3!) + (9/4)^4(1/4!) + (9/4)^5(1/4!) + \dots] = 1$
 $P_0[1 + (9/4) + (9/4)^2(1/2!) + (9/4)^3(1/3!) + (9/4)^4(1/4!)\{1 + (9/4) + (9/4)^2 + \dots\}] = 1$
 $P_0[1 + (9/4) + (9/4)^2(1/2!) + (9/4)^3(1/3!) + (9/4)^4(1/4!)(1/(1 - 9/16))] = 1$
 $P_0[1 + 2.25 + 2.53 + 1.89 + 2.44] = 1$
 $P_0 = 0.099$

Here we can see that there will be no queue if the number of patients are less than the number of physicians and in that case length of the queue will be zero. If there number of patients arriving exceeds the number of physicians or servers then a queue will form. Now we want to know the length of the queue.

$$L_q = \left[\frac{\rho^{c+1}}{(c-1)!(c-\rho)^2} \right] p_0 = 5.24$$

$$L_s = L_q + \rho = 5.24 + 2.25 = 7.49$$

Moreover to calculate waiting time in the queue and waiting time in the system we use little's law and we get the following euations
 $W_q = L_q/\lambda = 5.24/9 = 0.582$ hours = 35 minutes approximately.
 $W_s = L_s/\lambda = 7.49/9 = 0.832$ = 50 minutes approximately.

Results and conclusions:

On the basis of data analyzed this study reveals that there are long queues in the Emergency Department and patients needs to wait for hours. Queuing theory can act as a way to minimize the length of the queues and waiting time of the patients so that the service can be made faster.. Increasing the number of beds and consequently other resources appropriately can solve the problem up to a large extent.

Especially those who can't afford the highly expensive service of private hospitals will get benefits if we would be able to improve the quality of the service. Moreover Hospitals should share their past experiences in order to improve emergency plans and motivating hospital management teams to find better solutions for reducing the risk of accidents.

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